

Admission Exam PPG-EM – 2024/1^o sem

Candidate Name:
ID:
Date:
Signature:

Instructions

- 1) The exam consists of 10 questions, and the candidate must choose 5 questions to solve. In case the candidate solves a greater number of questions, only the first 5 will be considered;
- 2) All questions have the same value (2.0 points for each question);
- 3) It is not allowed to consult any type of material;
- 4) The use of simple (non-programmable) electronic calculators is permitted;
- 5) All answers must be justified;
- 6) The duration of the exam is 3 hours.

For the exclusive use of examiners

QUESTIONS SCORES									
Q1		Q3		Q5		Q7		Q9	
Q2		Q4		Q6		Q8		Q10	

TOTAL SCORE

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QUESTION 1: (Linear Algebra)

Calculate the value of the exponent n for the expression below, which involves the determinant of the product of matrix \mathbf{A} and its transpose.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}, \quad |\mathbf{A} \times \mathbf{A}^T| = 2^n$$

Justify your answer.

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QUESTION 2: (Differential and Integral Calculus)

- a) Calculate the expression for the volume of a cone with height **H** and radius of the base **R** using integration.
- b) Calculate the height of the cylinder with maximum volume that can be inscribed into a cone with height **H** and radius of the base **R**.

Justify your answer.

QUESTION 3: (Computation - continuation)

Write a program in a high-level language (C, C++ or Python) that implements a recursive function to calculate the n^{th} term of the Fibonacci sequence. The Fibonacci sequence is defined as follows:

$$F(n) = F(n-1) + F(n-2)$$

with the initial values:

$$F(0) = 0$$

$$F(1) = 1$$

The function must accept an n argument and return the n^{th} term of the Fibonacci sequence.

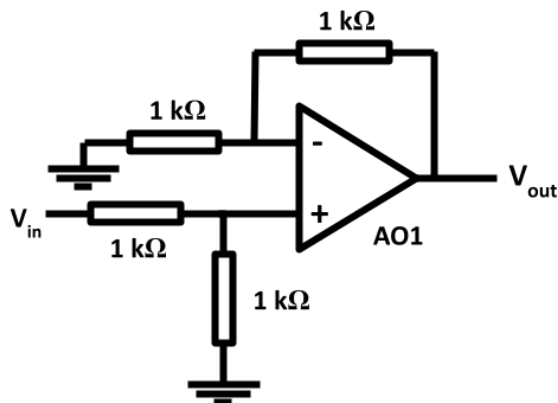
Note: Be sure to include explanatory comments in the code to help you understand how the implemented function works.

b) Write an example program using the recursive function, calculating and displaying the n^{th} term for a specific value of n , for example, $n = 6$.

Justify your answer.

QUESTION 4: (Electronics)

According to the circuit below, calculate the gain (V_{out}/V_{in}) and input resistance (R_{in}) values for the non-inverting amplifier stage, based on an operational amplifier. To do this, consider the characteristics of the components as being ideal.



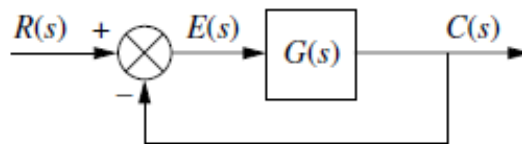
Justify your answer.

QUESTION 5: (Control Systems)

What is the range of K values that guarantee the system's closed-loop stability?

Given that:

$$G(s) = \frac{K}{(s-1)(s+3)}$$



Justify your answer.

QUESTION 6: (Materials)

Temperature has a significant effect on almost all properties of a material.

a) Draw schematically the influence of increasing temperature on the yield and ultimate strengths and on the ductility of a material.

b) Consider the following materials: low-carbon steel, high-carbon steel, high alloy steel, and ceramics.

Draw schematically the influence of increasing temperature on the hardness of these materials.

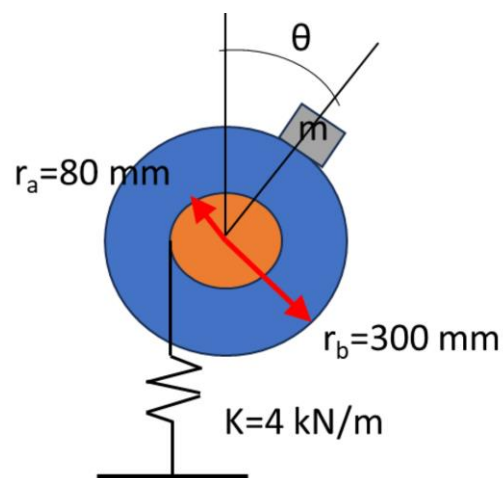
Justify your answer.

QUESTION 7: (General Mechanics)

A block of mass m is attached to a disk with a radius of $r_b = 300$ mm. A spring with a constant $K = 4$ kN/m is attached to the inner disk with a radius of $r_a = 80$ mm. The entire assembly can rotate without friction around the center of the disks, and there is no slipping between the disks or between the mass and the outer disk. When $\theta = 0^\circ$, the spring is in its natural (undisturbed) condition. Determine:

- The mass m of the block for which $\theta = 45^\circ$ becomes one of the equilibrium positions of the system.
- Discuss if the given system has unstable equilibrium points.

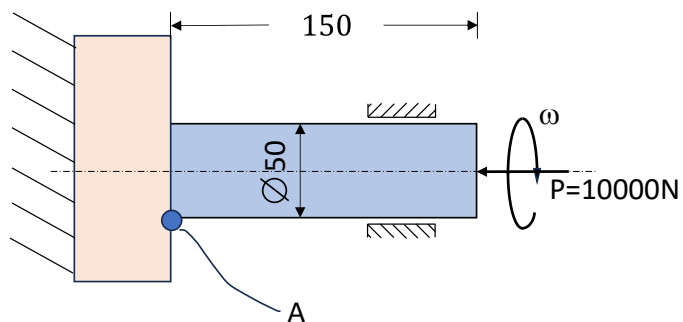
OBS: Gravity acceleration $g = 9.8$ m/s².



Justify your answer.

QUESTION 8: (Solid Mechanics)

A shaft of constant section, carried by an axial load, rotates at constant speed on a surface that exhibits a coefficient of friction equal to 0.25 between the supporting surfaces. Considering that the force is uniformly distributed, calculate the state of stress at point A.



Justify your answer.

QUESTION 9: (Thermodynamics)

In a heat treatment process, a 4.12 kg piece of solid metal ($c_m = 0.6 \text{ kJ}/(\text{kg}\cdot\text{K})$), initially at 1050 K, is tempered in a tank containing 220 kg of water ($c_{ag} = 4.2 \text{ kJ}/(\text{kg}\cdot\text{K})$), initially at 300 K. The heat exchanged between the contents of the tank and its external surroundings is negligible. Considering that the specific heats are constant, uniform properties for each material and that the effects of volume changes are negligible (both substances incompressible), determine:

- (a) The final equilibrium temperature of the composite system, that is, water plus metal, in K (neglect temperature variations of the tank wall material).
- (b) The change in entropy of metal plus water system, in kJ/kg. Remember that the entropy variation should be computed by integrating the respective expressions from states 1 to 2.

Formulae:

Energy conservation: $\delta Q = dE + \delta W$ or ${}_1Q_2 = \Delta E + {}_1W_2$

Entropy variation or change: $TdS = dU + PdV$ or $dS = (dU/T) + (P/T)dV$

Where: W - work [J]; Q - heat [J]; E - total energy [J]; S - Entropy [kJ/kg]; c - specific heat [J/(kg·K)]; m - mass [kg]; T - temperature [K]; U - total internal energy [J]; V - volume [m³].

Note that $dE = dU + dEC + dEP$, being EC the total kinetic energy [J] and EP the total potential energy [J].

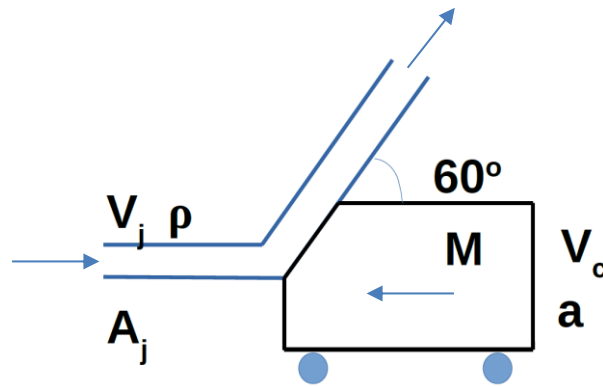
For an incompressible substance $dU = mcdT$.

Subscripts: m - metal, ag - water, 1 - state 1, 2 - state 2.

Justify your answer.

QUESTION 10: (Fluid Mechanics)

Consider the sketch. In a situation in which the car speed (V_c) is half the jet speed (V_j), what is the acceleration (\mathbf{a}) of the car? Consider: jet section area, A_j , jet fluid density, ρ , car mass, M .



Formulae:

$$\frac{d}{dt} \int_{VC} \rho dV + \int_{SC} \rho (\vec{V} \cdot \vec{n}) dA = 0$$

$$\frac{d}{dt} \int_{VC} \rho \vec{V} dV + \int_{SC} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = \vec{F}$$

Justify your answer.